A General Small-Signal Series Impedance Extraction Technique

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***Abstract*—A new technique for extracting the series inductances and resistances in a small-signal equivalent circuit is presented. The technique does not rely on approximation and should therefore be as accurate as the measured data. The technique can also be used to extract the intrinsic parameters if they are not easily achieved using other methods. The method is exemplified with a microwave LDMOS transistor.**

***Index Terms*—Equivalent circuits, impedance measurement, microwave transistors, parameter estimation.**

I. Introduction

When dealing with small-signal equivalent circuits, the extraction of parameters is of greatest importance. The first problem is to de-embed the pad parasitics. These parasitics include everything between the calibration plane and the intrinsic device. There are several ways [1]–[3] to do this, and they all have their pros and cons in terms of simplicity, accuracy, and the types of devices that are characterized.

Then, the problem is to extract the series impedances, which are the gate, drain, and source resistances and their corresponding inductances. There are several different strategies to do that. A method originating from the ColdFET method used for MESFETs assumes that the intrinsic device is purely capacitive when the applied biases are zero [4]. In that method, the series resistances are supposed to be independent of bias and are extracted only at one bias condition. Other methods take the intrinsic part into account and extrapolates the curves to extract the series impedances [1], [5], which can be done for each bias point. This is made possible by approximating the intrinsic part until it is simple enough.

In this paper, we present a method to extract the series impedances exactly. Exactly should be interpreted as no approximations and assumptions except the choice of small-signal equivalent circuit (which is not a trivial choice). The method can be used on a variety of circuit models, primarily FET models, and we will demonstrate the method by applying it on a microwave LDMOS transistor.

II. Theory

For any small-signal equivalent circuit, the analytical expression for the Z-parameters can be written as in (1)–(4). Note that the left hand side sometimes must be multiplied by s so that $b_0 = 1$ and $a_0 \neq 0$ ($s = j\omega$, the complex frequency)

$$sz_{11} = \frac{a_0^{11} + sa_1^{11} + \cdots + s^{n_{11}} a_{n_{11}}^{11}}{1 + sb_1 + \cdots + s^m b_m} \tag{1}$$

$$z_{12} = \frac{a_0^{12} + sa_1^{12} + \cdots + s^{n_{12}} a_{n_{12}}^{12}}{1 + sb_1 + \cdots + s^m b_m} \tag{2}$$

$$sz_{21} = \frac{a_0^{21} + sa_1^{21} + \cdots + s^{n_{21}} a_{n_{21}}^{21}}{1 + sb_1 + \cdots + s^m b_m} \tag{3}$$

$$z_{22} = \frac{a_0^{22} + sa_1^{22} + \cdots + s^{n_{22}} a_{n_{22}}^{22}}{1 + sb_1 + \cdots + s^m b_m}. \tag{4}$$

Each coefficient can be expressed in terms of the different parameters in the equivalent circuit ($s^{n_{11}} a_{n_{11}}^{11}$ should be interpreted as s to the power of n_{11} times the coefficient $a_{n_{11}}^{11}$).

By extraction of the coefficients using the least square method, it is possible to obtain a equation system with more equations than unknowns. This makes it possible to extract several parameters without any approximations or assumptions. A slight problem is that the new equation system is overdetermined and, thus, care has to be taken with how to extract the parameters.

In order to calculate the a- and b-coefficients using the least square method, the following matrix system (5) has to be evaluated:

$$X\mathbf{a} = \mathbf{y} \tag{5}$$

where X is a ($p \times p$)-matrix, $\mathbf{a}$ and $\mathbf{y}$ are vectors with length p, where $p = n_{11} + n_{12} + n_{21} + n_{22} + m$. The matrices can be expressed using sub-matrices as in (6)–(8)

$$X = \begin{bmatrix} X_{A,11} & 0 & 0 & 0 & X_{C,11}^t \\ 0 & X_{A,12} & 0 & 0 & X_{C,12}^t \\ 0 & 0 & X_{A,21} & 0 & X_{C,21}^t \\ 0 & 0 & 0 & X_{A,22} & X_{C,22}^t \\ X_{C,11} & X_{C,12} & X_{C,21} & X_{C,22} & X_B \end{bmatrix} \tag{6}$$

$$\mathbf{a} = [\mathbf{a_{11}} \quad \mathbf{a_{12}} \quad \mathbf{a_{21}} \quad \mathbf{a_{22}} \quad \mathbf{a_B}]^t \tag{7}$$

$$\mathbf{y} = [\mathbf{y_{11}} \quad \mathbf{y_{12}} \quad \mathbf{y_{21}} \quad \mathbf{y_{22}} \quad \mathbf{y_B}]^t. \tag{8}$$

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The structure of the sub-matrices follows below, where all sums are over the measured frequency sweep with length N

$$X_{A,ij} = \begin{bmatrix} N & \Sigma s & \cdots & \Sigma s^{n_{ij}} \\ \Sigma s & \Sigma s^2 & \cdots & \Sigma s^{n_{ij}+1} \\ \vdots & \vdots & \ddots & \vdots \\ \Sigma s^{n_{ij}} & \Sigma s^{n_{ij}+1} & \cdots & \Sigma s^{2n_{ij}} \end{bmatrix} \quad (9)$$

$$X_B = \begin{bmatrix} \Sigma s^2(\Sigma_{ij} z_{ij}^2) & \cdots & \Sigma s^{m+1}(\Sigma_{ij} z_{ij}^2) \\ \vdots & \ddots & \vdots \\ \Sigma s^{m+1}(\Sigma_{ij} z_{ij}^2) & \cdots & \Sigma s^{2m}(\Sigma_{ij} z_{ij}^2) \end{bmatrix} \quad (10)$$

$$X_{C,ij} = \begin{bmatrix} \Sigma s z_{ij} & \cdots & \Sigma s^{n_{ij}} z_{ij} \\ \vdots & \ddots & \vdots \\ (\Sigma s^m z_{ij}) & \cdots & (\Sigma s^{n_{ij}+m} z_{ij}) \end{bmatrix} \quad (11)$$

$$\mathbf{a_{ij}} = [\,a_0^{ij} \quad \cdots \quad a_{n_{ij}}^{ij}\,]^{\mathbf{t}} \quad (12)$$

$$\mathbf{a_B} = [\,-b_1 \quad \cdots \quad -b_m\,]^{\mathbf{t}} \quad (13)$$

$$\mathbf{y_{ij}} = [\,\Sigma z_{ij} \quad \cdots \quad \Sigma s^{n_{ij}} z_{ij}\,]^{\mathbf{t}} \quad (14)$$

$$\mathbf{y_B} = [\,\Sigma s(\Sigma_{ij} z_{ij}^2) \quad \cdots \quad \Sigma s^m(\Sigma_{ij} z_{ij}^2)\,]^{\mathbf{t}}. \quad (15)$$

The following interpretations of (6)–(15) should be made:

$$\Sigma s^2 = \sum_{k=1}^{N} s_k^2 \quad (16)$$

$$\Sigma_{ij} z_{ij} = \sum_{i=1}^{2} \sum_{j=1}^{2} z_{ij} = z_{11} + z_{12} + z_{21} + z_{22} \quad (17)$$

$$\Sigma_{ij} z_{ij}^2 = z_{11}^2 + z_{12}^2 + z_{21}^2 + z_{22}^2 \quad (18)$$

$$0 = \text{zero matrix of appropriate size.} \quad (19)$$

Applying the above calculations on the measured data z_k will yield complex coefficients a and b. What we actually want is that the coefficients should be real values. Thus, the real and imaginary parts of X and $\mathbf{y}$ should be treated separately. The easiest way to do this is to calculate the complex matrices X and $\mathbf{y}$ and the rewrite them as

$$X_{ri} = \Re(X) + \Im(X) \quad (20)$$

$$\mathbf{y}_{ri} = \Re(\mathbf{y}) + \Im(\mathbf{y}) \quad (21)$$

$$\Rightarrow$$

$$\mathbf{a} = (X_{ri})^{-1}\mathbf{y}_{ri}. \quad (22)$$

III. Results

The method is applied on a microwave LDMOS transistor [6] ($W = 1$ mm) and the small-signal equivalent circuit in Fig. 1 is used. Setting up the analytical expressions for the Z-parameters as in (1)–(4) gives that $n_{11} = n_{21} = 3$, $n_{12} = n_{22} = 2$ and $m = 1$.

The X-matrix and the $\mathbf{y}$-vector are arranged and the a and b parameters are calculated using (22). Examining the analytical expression for the a and b parameters reveals that the series inductances can be calculated using

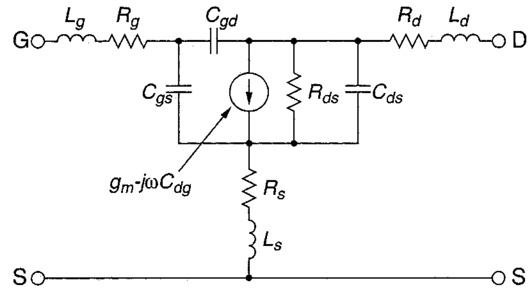

Fig. 1. Example of equivalent circuit for a microwave LDMOS transistor.

$$L_g + L_s = a_3^{11}/b_1 \quad (23)$$

$$L_s = a_2^{12}/b_1 = a_3^{21}/b_1 \quad (24)$$

$$L_d + L_s = a_2^{22}/b_1. \quad (25)$$

Notice that $a_2^{12} = a_3^{21}$. If they are unequal, which probably is the case, they should be replaced with their mean value. This can be done directly when setting up the least square matrices, but the result will be the same as taking the mean value. The simplicity of the inductance extraction is independent of the chosen equivalent circuit. Calculating the series inductances yields that $L_g = 91$ pH, $L_d = 74$ pH and $L_s = 0$ pH.

Rewriting the analytical expression for e.g., z_{12} as in (26) yields a new set of a-coefficients that can be calculated using (27)–(29) and thus eliminating L_s. This is done for all z_{ij} and thus all inductances are eliminated

$$z_{12} = \frac{a_0 + a_1 s + a_2 s^2}{1 + b_1 s} = sL_s + \frac{a_0' + a_1' s}{1 + b_1 s} \quad (26)$$

$$a_2' = a_2 - L_s b_1 = 0 \quad (27)$$

$$a_1' = a_1 - L_s \quad (28)$$

$$a_0' = a_0. \quad (29)$$

The new set of a'-coefficients are used to calculate the series resistances. The series resistances can be calculated using similar relations that the ones used for the inductances. The results from the resistance extraction yields: $R_g = 1.8\ \Omega$, $R_d = 5.4\ \Omega$ and $R_s = 2.5\ \Omega$. The series inductances and resistances can then be de-embedded from the measured Z-parameters for the extrinsic device by subtracting the series impedance sub-Z-matrix. The intrinsic parameters are then extracted using direct extraction from the intrinsic Y-parameters [1], [4], [5]. A comparison of the measured and modeled S-parameters are shown in Fig. 2.

IV. Discussion

The method described in this letter is shown to give accurate results for the series impedances. The strength of the method is that no approximations or assumptions about the intrinsic part has to be made in order to extract the series impedances. It should also be pointed out that unlike optimization methods

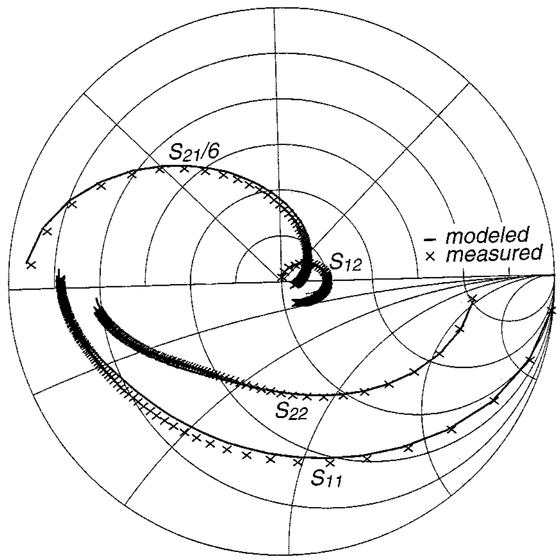

Fig. 2. Measured and modeled S-parameters for a microwave LDMOS transistor. Notice that the S_{21}-parameters are divided by six to fit in the plot.

this method does not require starting values of the parameters, which makes this a direct extraction method.

If the intrinsic part of the equivalent circuit is not as simple as in the example, it might be difficult to utilize direct extraction, since there are more unknowns than measured parameters. It is then possible to use the proposed technique for the intrinsic part to achieve more parameters than unknowns. If the equation system is overdetermined, some caution has to be taken in order to solve it. It might also be possible to extract the pad parasitics for a lumped representation of the pads.

In a large part of all published small-signal equivalent circuits, the transconductance is expressed with a phase parameter τ, $g_m = g_{m0}e^{-j\omega\tau}$. The proposed technique does not handle the sinusoidal functions that τ introduces without approximations. The most suitable approximation then would be to use a Taylor expansion of $e^{-j\omega\tau}$ with enough terms in the expansion to cover the region of validity. Another approach would be to use rational polynomials for g_m [7], which is fully covered by this method.

Using this method directly will get the best fit over the whole frequency range. To improve the extraction, there are several aspects that should be considered. The measured data can be weighted according to the uncertainty of the measurement equipment, so that more certain data are weighted heavier. The inductances, for example, that have their significance at higher frequencies should be extracted from the upper part of the frequency sweep.

V. Conclusion

We have presented a general method for extracting the series impedances in a small-signal equivalent circuit. The extraction technique handles arbitrary equivalent circuits and can also be extended to extract the intrinsic parameters. The technique is based on the least square method.

The strengths of the method are that no approximations or assumptions have to be made and that the method is a direct extraction method.

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